

SAMPLE SOLUTIONS FOR *DERIVATIVES MARKETS*

Question #1
Answer is D

If the call is at-the-money, the put option with the same cost will have a higher strike price. A purchased collar requires that the put have a lower strike price. **(Page 76)**

Question #2
Answer is C

$66.59 - 18.64 = 500 - K \exp(-0.06)$ for $K = 480$ **(Page 69)**

Question #3
Answer is D

The accumulated cost of the hedge is $(84.30 - 74.80) \exp(.06) = 10.09$.

Let x be the market price.

If $x < 0.12$ the put is in the money and the payoff is $10,000(0.12 - x) = 1,200 - 10,000x$.

The sale of the jalapenos has a payoff of $10,000x - 1,000$ for a profit of $1,200 - 10,000x + 10,000x - 1,000 - 10.09 = 190$.

From 0.12 to 0.14 neither option has a payoff and the profit is $10,000x - 1,000 - 10.09 = 10,000x - 1,010$.

If $x > 0.14$ the call is in the money and the payoff is $-10,000(x - 0.14) = 1,400 - 10,000x$.

The profit is $1,400 - 10,000x + 10,000x - 1,000 - 10.09 = 390$.

The range is 190 to 390. **(Pages 33-41)**

Question #4
Answer is B

The present value of the forward prices is $10,000(3.89)/1.06 + 15,000(4.11)/1.065^2 + 20,000(4.16)/1.07^3 = 158,968$. Any sequence of payments with that present value is acceptable. All but B have that value. **(Page 248)**

Question #5
Answer is E

If the index exceeds 1,025, you will receive $x - 1,025$. After buying the index for x you will have spent 1,025. If the index is below 1,025, you will pay $1,025 - x$ and after buying the index for x you will have spent 1,025. One way to get the cost is to note that the forward price is $1,000(1.05) = 1,050$. You want to pay 25 less and so must spend $25/1.05 = 23.81$ today. **(Page 112)**

Question #6

Answer is E

In general, an investor should be compensated for time and risk. A forward contract has no investment, so the extra 5 represents the risk premium. Those who buy the stock expect to earn both the risk premium and the time value of their purchase and thus the expected stock value is greater than $100 + 5 = 105$. **(Page 140)**

Question #7

Answer is C

All four of answers A-D are methods of acquiring the stock. The prepaid forward has the payment at time 0 and the delivery at time T . **(Pages 128-129)**

Question #8

Answer is B

Only straddles use at-the-money options and buying is correct for this speculation. **(Page 78)**

Question #9

Answer is D

This is based on Exercise 3.18 on **Page 89**. To see that D does not produce the desired outcome, begin with the case where the stock price is S and is below 90. The payoff is $S + 0 + (110 - S) - 2(100 - S) = 2S - 90$ which is not constant and so cannot produce the given diagram. On the other hand, for example, answer E has a payoff of $S + (90 - S) + 0 - 2(0) = 90$. The cost is $100 + 0.24 + 2.17 - 2(6.80) = 88.81$. With interest it is 93.36. The profit is $90 - 93.36 = -3.36$ which matches the diagram.

Question #10

Answer is D

- [rationale-a] True, since forward contracts have no initial premium
- [rationale-b] True, both payoffs and profits of long forwards are opposite to short forwards.
- [rationale-c] True, to invest in the stock, one must borrow 100 at $t=0$, and then pay back $110 = 100*(1+.1)$ at $t=1$, which is like buying a forward at $t=1$ for 110.
- [rationale-d] False, repeating the calculation shown above in part c), but with 10% as a continuously compounded rate, the stock investor must now pay back $100*e^{.1} = 110.52$ at $t=1$; this is more expensive than buying a forward at $t=1$ for 110.00.
- [rationale-e] True, the calculation would be the same as shown above in part c), but now the stock investor gets an additional dividend of 3.00 at $t=.5$, which the forward investor does not receive (due to not owning the stock until $t=1$).

[This is based on Exercise 2-7 on p.54-55 of McDonald]

[McDonald, Chapter 2, p.21-28]

Question #11

Answer is C

- Solution: The 35-strike call has future cost (at $t=1$) of $9.12*(1+.08) = 9.85$
The 40-strike call has future cost (at $t=1$) of $6.22*(1+.08) = 6.72$
The 45-strike call has future cost (at $t=1$) of $4.08*(1+.08) = 4.41$
- If $S_1 < 35$, the profits of the 3 calls, respectively, are -9.85, -6.72, and -4.41.
If $35 < S_1 < 40$, the profits of the 3 calls, respectively, are $S_1 - 44.85$, -6.72, and -4.41.
If $40 < S_1 < 45$, the profits of the 3 calls, respectively, are $S_1 - 44.85$, $S_1 - 46.72$, and -4.41.
If $S_1 > 45$, the profits of the 3 calls, respectively, are $S_1 - 44.85$, $S_1 - 46.72$, and $S_1 - 49.41$.
- The cutoff points for when the relative profit ranking of the 3 calls change are:
 $S_1 - 44.85 = -6.72$, $S_1 - 44.85 = -4.41$, and $S_1 - 46.72 = -4.41$, yielding cutoffs of 38.13, 40.44, and 42.31.
- If $S_1 < 38.13$, the 45-strike call has the highest profit, and the 35-strike call the lowest.
If $38.13 < S_1 < 40.44$, the 45-strike call has the highest profit, and the 40-strike call the lowest.
If $40.44 < S_1 < 42.31$, the 35-strike call has the highest profit, and the 40-strike call the lowest.
If $S_1 < 42.31$, the 35-strike call has the highest profit, and the 45-strike call the lowest.
- We are looking for the case where the 35-strike call has the highest profit, and the 40-strike call has the lowest profit, which occurs when S_1 is between 40.44 and 42.31.

[This is based on Exercise 2-13 on p.55-56 of McDonald]

[McDonald, Chapter 2, p.33-37]

Question #12

Answer is B

Solution: The put premium has future value (at $t=.5$) of $74.20 * (1+(.04/2)) = 75.68$. Then, the 6-month profit on a long put position is: $\max(1,000-S_{.5},0)-75.68$. Correspondingly, the 6-month profit on a short put position is $75.68-\max(1,000-S_{.5},0)$. These two profits are opposites (naturally, since long and short positions have opposite payoff and profit). Thus, they can only be equal if producing 0 profit. 0 profit is only obtained if $75.68 = \max(1,000-S_{.5},0)$, or $1,000-S_{.5} = 75.68$, or $S_{.5} = 924.32$.

[McDonald, Chapter 2, p.39-42]

Question #13

Answer is D

Solution: Buying a call, in conjunction with a short position in a stock index, is a form of insurance called a cap. Answers (A) and (B) are incorrect because they relate to a floor, which is the purchase of a put to insure against a long position in a stock index. Answer (E) is incorrect because it relates to writing a covered call, which is the sale of a call along with a long position in the stock index, so that the investor is selling rather than buying insurance. Note that a cap can also be thought of as 'buying' a covered call. Now, let's calculate the profit:

2-year profit = payoff at time 2 – the future value of the initial cost to establish the position
 $= (-75 + \max(75-60,0)) - (-50 + 10)*(1+.03)^2 = -75+15+40*(1.03)^2 = 42.44-60 = -17.56$.
Thus, we've lost more from holding the short position in the index (since the index went up) than we've gained from owning the long call option.

[McDonald, Chapter 3, p.59-65]

Question #14

Answer is A

Solution: This consists of standard applications of the put-call parity equation on p.69. Let C be the price for the 40-strike call option. Then, $C + 3.35$ is the price for the 35-strike call option. Similarly, let P be the price for the 40-strike put option. Then, $P - x$ is the price for the 35-strike put option, where x is what we're trying to find. Using put-call parity, we have:

$$(C + 3.35) + 35 * e^{-.02} - 40 = P - x \quad (\text{this is for the 35-strike options})$$

$$C + 40 * e^{-.02} - 40 = P \quad (\text{this is for the 40-strike options})$$

$$\text{Subtracting the first equation from the second, } 5 * e^{-.02} - 3.35 = x = 1.55.$$

[McDonald, Chapter 3, p.68-69]

Question #15

Answer is C

Solution: The initial cost to establish this position is $5*2.78 - 3*6.13 = -4.49$. Thus, you are receiving 4.49 up front. This grows to $4.49*e^{.08*.25} = 4.58$ after 3 months. Then, the following payoff/profit table can be constructed at $T=.25$ years:

S_T :	$5*\max(S_T - 40, 0)$	$- 3*\max(S_T - 35, 0)$	$+ 4.58 = \text{Profit}$
$S_T < 35$	0	- 0	$+ 4.58 = 4.58$
$35 \leq S_T \leq 40$	0	- $3*(S_T - 35)$	$+ 4.58 = 109.58 - 3S_T$
$S_T > 40$	$5*(S_T - 40)$	- $3*(S_T - 35)$	$+ 4.58 = 2S_T - 90.42$

Thus, the maximum profit is unlimited (as S_T increases beyond 40, so does the profit)
 Also, the maximum loss is 10.42 (occurs at $S_T = 40$, where profit = $109.58 - 120 = -10.42$)

[Notes] The above problem is an example of a ratio spread.

[McDonald, Chapter 3, p.73]

Question #16

Answer is D

Solution: The 'straddle' consists of buying a 40-strike call and buying a 40-strike put. This costs $2.78 + 1.99 = 4.77$ at $t=0$, and grows to $4.77*e^{.02} = 4.87$ at $t=.25$. The 'strangle' consists of buying a 35-strike put and a 45-strike call. This costs $0.44 + 0.97 = 1.41$ at $t=0$, and grows to $1.41*e^{.02} = 1.44$ at $t=.25$. For $S_T < 40$, the 'straddle' has a profit of $40 - S_T - 4.87 = 35.13$, and for $S_T \geq 40$, the 'straddle' has a profit of $S_T - 40 - 4.87 = 44.87$. For $S_T < 35$, the 'strangle' has a profit of $35 - S_T - 1.44 = 33.56$, and for $S_T > 45$, the 'strangle' has a profit of $S_T - 45 - 1.44 = 46.44$. However, for $35 \leq S_T \leq 45$, the 'strangle' has a profit of -1.44 (since both options would not be exercised). Comparing the payoff structures between the 'straddle' and 'strangle,' we see that if $S_T < 35$ or if $S_T > 45$, the 'straddle' would outperform the 'strangle' (since $35.13 > 33.56$, and since $-44.87 > -46.44$). However, if $35 \leq S_T \leq 45$,

we can solve for the two cutoff points for S_T , where the 'strangle' would outperform the 'straddle' as follows:
 $-1.44 > 35.13 - S_T$, and $-1.44 > S_T - 44.87$. The first inequality gives $S_T > 36.57$, and the second inequality gives $S_T < 43.43$. Thus, $36.57 < S_T < 43.43$.

[McDonald, Chapter 3, p.78-80]

Question # 17

Answer is B

[rationale I] Yes, since Strategy I is a bear spread using calls, and bear spreads perform better when the prices of the underlying asset goes down.

[rationale II] Yes, since Strategy II is also a bear spread – it just uses puts instead!

[rationale III] No, since Strategy III is a box spread, which has no price risk; thus, the payoff is the same ($1,000 - 950 = 50$), no matter what the price of the underlying asset.

[Note]: An alternative, but much longer, solution is to develop payoff tables for all 3 strategies.

[McDonald, Chapter 3, p.70-73]

Question #18

Answer is B

Solution: First, let's calculate the expected one-year profit without using the forward. This would be $.2*(700+150-750) + .5(700+170-850) + .3*(700+190-950) = 20 + 10 - 18 = 12$. Next, let's calculate the expected one-year profit when buying the 1-year forward for 850. This would be $1*(700+170-850) = 20$. Thus, the expected profit increases by $20 - 12 = 8$ as a result of using the forward.

[This is based on Exercise 4-7 on p.122 of McDonald]

[McDonald, Chapter 4, p.98-100]

Question #19

Answer is D

Solution: There are 3 cases, one for each row in the above probability table. For all 3 cases, the future value of the put premium (at $t=1$) = $100 * e^{.06} = 106.18$. In Case 1, the 1-year profit would be: $750 - 800 - 106.18 + \max(900-750,0) = -6.18$. In Case 2, the 1-year profit would be: $850 - 800 - 106.18 + \max(900-850,0) = -6.18$. In Case 3, the 1-year profit would be: $950 - 800 - 106.18 + \max(900-950,0) = 43.82$. Thus, the expected 1-year profit = $.7 * -6.18 + .3 * 43.82 = -4.326 + 13.146 = 8.82$.

[This is based on Exercise 4-3 on p.121 of McDonald]

[McDonald, Chapter 4, p.92-96]

Question # 20

Answer is B

Solution: This is an example of pricing a forward contract using discrete dividends. Thus, we need the future value of the current stock price minus the future value of each of the 12 dividends, where the valuation date is $T=3$. Thus, the valuation equation is: Forward price = $200 * e^{.04(3)} - [1.50 * e^{.04(2.75)} + 1.50 * 1.01 * e^{.04(2.5)} + 1.50 * (1.01)^2 * e^{.04(2.25)} + \dots + 1.50 * (1.01)^{12}] = 200 * e^{.12} - 1.50 * e^{.11} \{ [1 - (1.01 * e^{-.01})^{12}] / [1 - (1.01 * e^{-.01})] \}$, using the geometric series formula from interest theory. This simplifies numerically to $225.50 - 1.67442 * 11.99666 = 205.41$.

[This problem combines the material from interest theory and derivatives, although one could also simplify the above expression by brute force (instead of geometric series), since there are only 12 dividends to accumulate forward to $T=3$.]

[McDonald, Chapter 5, p.133-134]

Question #21

Answer is E

Solution: Here, the fair value of the forward contract is given by $S_0 * e^{(r-d)T} = 110 * e^{(.05-.02).5} = 110 * e^{.015} = 111.66$. This is 0.34 less than the observed price. Thus, one could exploit this arbitrage opportunity by selling the observed forward at 112 and buying a synthetic forward at 111.66, making $112 - 111.66 = 0.34$ profit.

[This is based on Exercise 5-8 on p.163-164 of McDonald]

[McDonald, Chapter 5, p.135-138]

Question #22

Answer is B

Solution: First, we must determine the present value of the forward contracts. On a per ton basis, this is: $1,600/1.05 + 1,700/(1.055)^2 + 1,800/(1.06)^3 = 4,562.49$.

Then, we must solve for the level swap price, which is labeled x below, as follows:

$$4,562.49 = x/1.05 + x/(1.055)^2 + x/(1.06)^3 = x*[1/1.05 + 1/(1.055)^2 + 1/(1.06)^3] = 2.69045*x.$$

Thus, $x = 4,562.49 / 2.69045 = 1,695.81$.

Thus, the amount he would receive each year is $50*1,695.81 = 84,790.38$.

[McDonald, Chapter 8, p.247-248]

Question #23

Answer is E

Solution: First, note that the notional amount and the future 1-year LIBOR rates (not given) do not factor into the calculation of the swap's fixed rate. All we need at the various zero-coupon bond prices $P(0, t)$ for $t=1,2,3,4,5$, along with the 1-year implied forward rates, which are given by $r_0(t-1, t)$, for $t=1,2,3,4,5$. These calculations are shown in the following table:

<u>t</u>	1	2	3	4	5
<u>P(0,t)</u>	$(1.04)^{-1}$ =.96154	$(1.045)^{-2}$ =.91573	$(1.0525)^{-3}$ =.85770	$(1.0625)^{-4}$ =.78466	$(1.075)^{-5}$ =.69656
<u>$r_0(t-1,t)$</u>	s_1 =.04000	$[1.045^2/1.04]-1$ =.05002	$[1.0525^3/1.045^2]-1$ =.06766	$[1.0625^4/1.0525^3]-1$ =.09307	$[1.075^5/1.0625^4]-1$ =.12649

Thus, the fixed swap rate = $R = [(.96154)*(.04)+...+(.69656)*(.12649)] / [.96154 + ... + .69656]$
 $= [.03846 + .04580 + .05803 + .07303 + .08811]/[.96154 + .91573 + .85770 + .78466 + .69656]$
 $= .30344 / 4.21619 = .07197 = 7.20\%$ (approximately).

[Note: This is much less calculation-intensive if you realize that the numerator (.30344) for R can also be obtained by taking $1 - P(0,n) = 1 - P(0,5) = 1 - .69656 = .30344$. Then, you would not need to calculate any of the implied forward rates!]

[McDonald, Chapter 8, p.255-258]

Question #24

Answer is D

- [rationale-a] True, hedging reduces the risk of loss, which is a primary function of derivatives.
- [rationale-b] True, derivatives can be used to hedge some risks that could result in bankruptcy.
- [rationale-c] True, derivatives can provide a lower-cost way to effect a financial transaction.
- [rationale-d] False, derivatives are often used to avoid these types of restrictions.
- [rationale-e] True, an insurance contract can be thought of as a hedge against the risk of loss.

[McDonald, Chapter 1, p.2-3]

Question #25

Answer is C

- [rationale-a] True, both types of individuals are involved in the risk-sharing process.
- [rationale-b] True, this is the primary reason reinsurance companies exist.
- [rationale-c] False, reinsurance companies share risk by issuing rather than investing in catastrophic bonds. In effect, they are ceding this excess risk to the bondholder.
- [rationale-d] True, it is diversifiable risk which is reduced or eliminated when risks are shared.
- [rationale-e] True, this is a fundamental idea underlying risk management and derivatives.

[McDonald, Chapter 1, p.5-6]

Question #26

Answer is B

- [rationale-I] True, the forward seller has unlimited exposure if the underlying asset's price increases.
- [rationale-II] True, the call issuer has unlimited exposure if the underlying asset's price rises.
- [rationale-III] False, the maximum loss on selling a put is $FV(\text{put premium}) - \text{strike price}$.

[McDonald, Chapter 2, p.43 (Table 2.4)]

Question #27

Answer is A

- [rationale-I] True, as prices go down, the value of holding a put option increases. Homeowner's insurance can be thought of as a put option.
- [rationale-II] False, returns from equity-linked CDs are zero if prices decline, but positive if prices rise. Thus, owners of these CDs benefit from rising prices.
- [rationale-III] False, a synthetic forward consists of a long call and a short put, both of which benefit from rising prices (so the net position also benefits as such).

[McDonald, Chapter 2, p.44-48]

Question #28

Answer is E

- [rationale-a] True, derivatives are used to shift income, thereby potentially lowering taxes.
- [rationale-b] True, as with taxes, the transfer of income lowers the probability of bankruptcy.
- [rationale-c] True, hedging can safeguard reserves, and reduce the need for external financing, which has both explicit (e.g. – fees) and implicit (e.g. – reputational) costs.
- [rationale-d] True, when operating internationally, hedging can reduce exchange rate risk.
- [rationale-e] False, a firm that credibly hedges will reduce the riskiness of its cash flows, and will be able to increase debt capacity, which will lead to tax savings, since interest is deductible.

[McDonald, Chapter 4, p.103-106]

Question #29

Answer is A

Solution: If S_0 is the price of the stock at time-0, then the following payments are required:

Outright purchase – payment at time 0 – amount of payment = S_0 .

Fully leveraged purchase – payment at time T – amount of payment = $S_0 * e^{rT}$.

Prepaid forward contract – payment at time 0 – amount of payment = $S_0 * e^{-dT}$.

Forward contract – payment at time T – amount of payment = $S_0 * e^{(r-d)T}$.

Since $r > d > 0$, it must be true that $S_0 * e^{-dT} < S_0 < S_0 * e^{(r-d)T} < S_0 * e^{rT}$.

Thus, the correct ranking is given by choice (A).

[McDonald, Chapter 5, p.127-134]

Question #30

Answer is C

- [rationale-a] True, marking to market is done for futures, and can lead to price differences relative to forward contracts.
- [rationale-b] True, futures are more liquid; in fact, if you use the same broker to buy and sell, your position is effectively cancelled.
- [rationale-c] False, forwards are more customized, and futures are more standardized.
- [rationale-d] True, because of the daily settlement, credit risk is less with futures (v. forwards).
- [rationale-e] True, futures markets, like stock exchanges, do have daily price limits.

[McDonald, Chapter 5, p.142]